ECE 330: Power Circuits and Electromechanics

Lecture 2

2019-08-28

Last Time

- Review of Phasors
- Use RMS instead of peak
- Calculating average (real) power

Today

- What is complex power?
- How do we calculate it?

Reactive (Q) and Complex (S) Power

$$P = V_{RMS} I_{RMS} \cos\left(\theta_v - \theta_i\right)$$

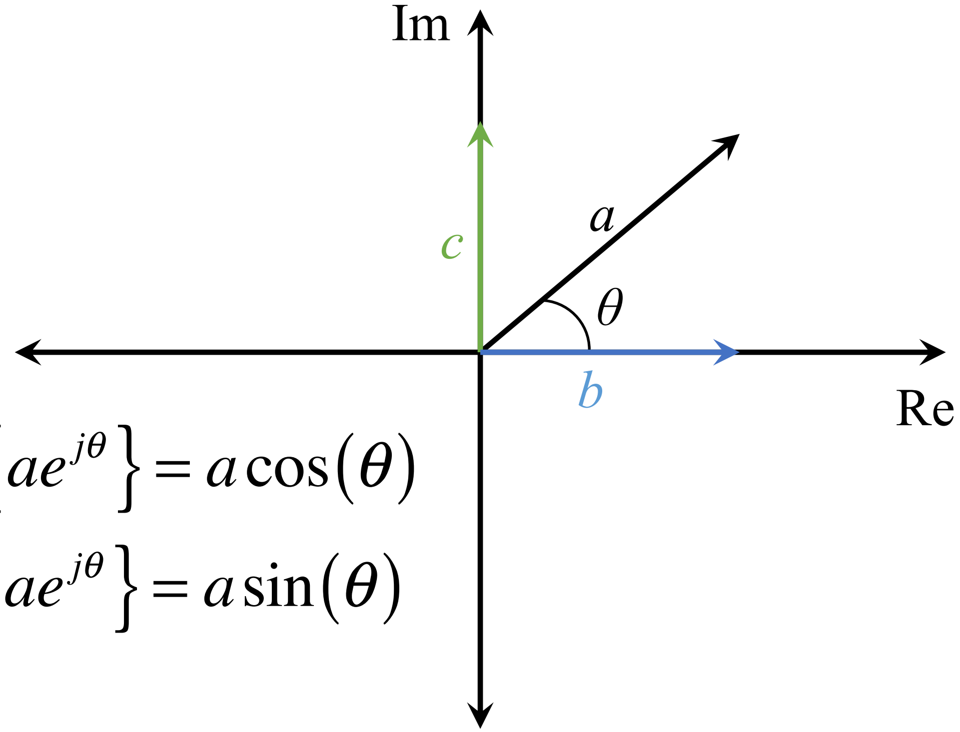
$$Q = V_{RMS} I_{RMS} \sin\left(\theta_v - \theta_i\right)$$

$$P = \mathrm{Re}\left\{V_{RMS} I_{RMS} e^{j(\theta_v - \theta_i)}\right\}$$

$$Q = \mathrm{Im}\left\{V_{RMS} I_{RMS} e^{j(\theta_v - \theta_i)}\right\}$$

$$P = \mathrm{Re}\left\{V_{RMS} e^{j\theta_v} I_{RMS} e^{-j\theta_i}\right\} = \mathrm{Re}\left\{\overline{V}\overline{I}^*\right\}$$

$$Q = \mathrm{Im}\left\{V_{RMS} e^{j\theta_v} I_{RMS} e^{-j\theta_i}\right\} = \mathrm{Im}\left\{\overline{V}\overline{I}^*\right\}$$



$$b = \mathrm{Re}\left\{a e^{j\theta}\right\} = a\cos(\theta)$$

$$c = \mathrm{Im}\left\{a e^{j\theta}\right\} = a\sin(\theta)$$

$$\overline{S} = \overline{V}\overline{I}^* = P + jQ$$

Units and Nomenclature

$$\left[\bar{S}\right] = \text{VA}$$

$$\left[P\right] = \text{W}$$

$$\left[Q\right] = \text{VAR}$$

$$\left\|\bar{S}\right\| = S = VI: \text{ apparent power}$$

$$\theta = \theta_v - \theta_i: \text{ power factor angle}$$

$$\cos(\theta) = \cos(\theta_v - \theta_i): \text{ power factor}$$

Different units to distinguish
different meanings.

Examples

$\bar{V} = 10\angle 15^\circ$ V, $\bar{I} = 5\angle -30^\circ$ A

Find P :

A) 50 W

B) 35.355 W

C) 48.30 W

D) 70.71 W

Examples

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C) -12.94 VAR

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Example

$$v(t) = \sqrt{2}(10)\cos\left(wt + 15^\circ\right)$$

$$i(t) = \sqrt{2}(7)\sin\left(\omega t + 75^\circ\right)$$

Find $\overline{S}$:

A) 70+j0 VA

B) 35-j60.62 VA

C) 60.62+j35 VA

D) 121.24+j70 VA

Example

$$v(t)=\sqrt{2}(10)\cos\left(wt+15^{\circ}\right)$$

$$i(t)=\sqrt{2}(7)\sin\left(\omega t+75^{\circ}\right)$$

Find $\overline{S}$:

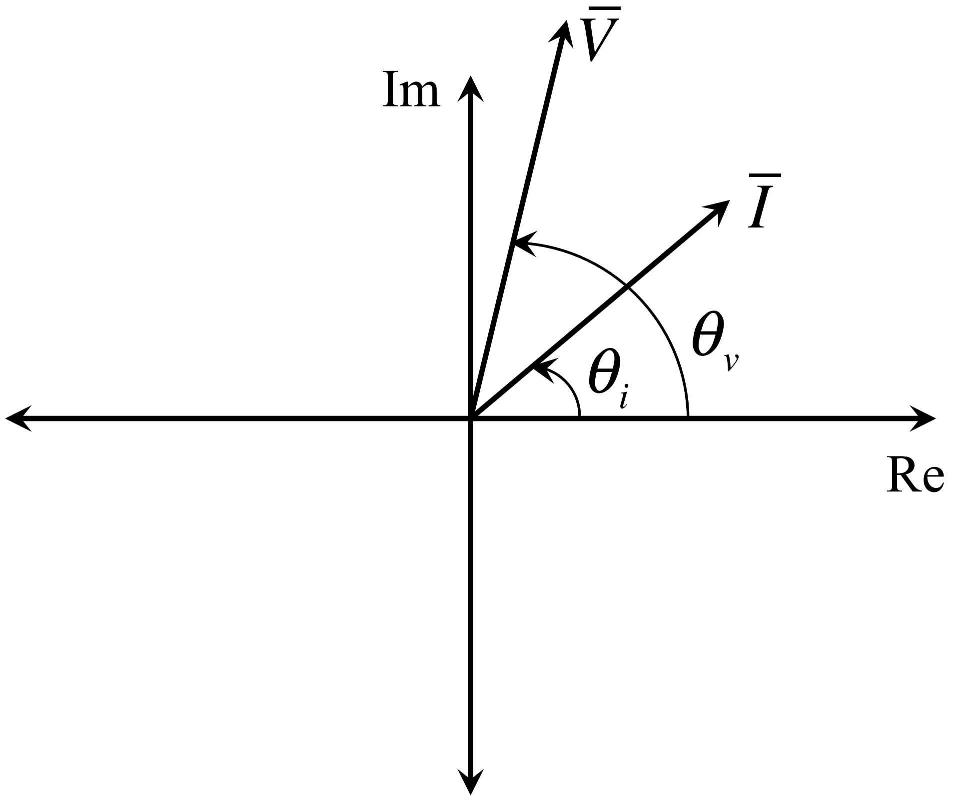
A) $70+j0$ VA

B) $35-j60.62$ VA

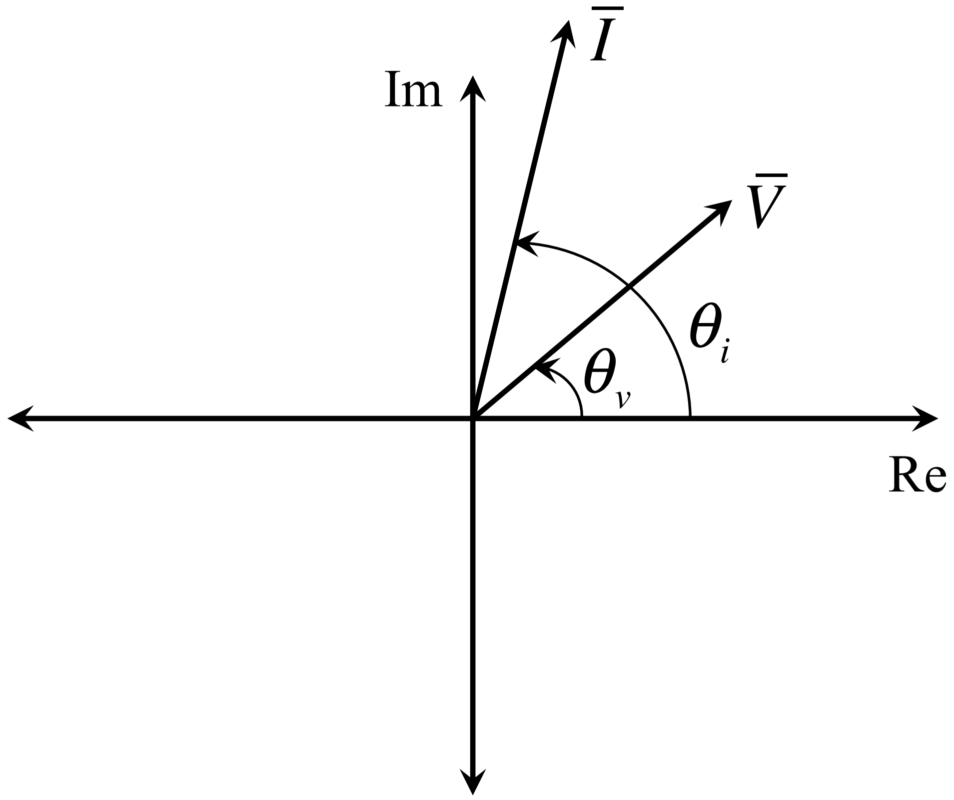
C) $60.62+j35$ VA

D) $121.24+j70$ VA

Recall: cosine is an even function $\cos(x)=\cos(-x)$



Current lags the voltage:
Lagging PF

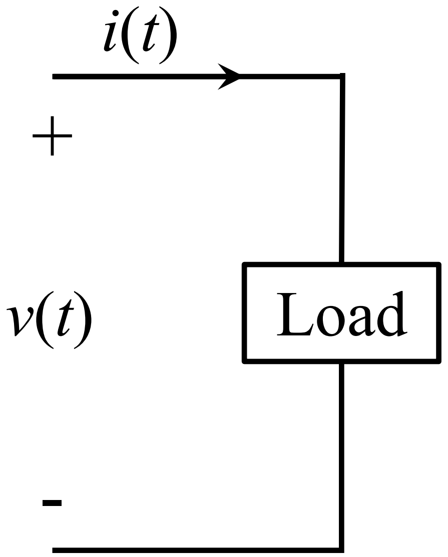


Current leads the voltage:
Leading PF

Note:

- PF is only defined for angles between -90 and +90 degrees
- Talking about the power consumed by the load
- If P is negative, this means power is generated by load, not consumed
- To correct, flip the direction for current definition.

Another way to view complex power



$\bar{Z} = R + jX$

$$\bar{V} = \overline{ZI}$$

$$\bar{S} = \overline{VI}^* = \bar{Z}\left(\overline{II}^*\right) = \bar{Z}I^2$$

$$\bar{S} = I^2R + jI^2X = P + jQ$$

P is power dissipated by resistors

Q is power dissipated by reactances

Can also show that angle on impedance is power factor angle

How do leading and lagging PF correspond to circuit?

$$v = L\frac{di}{dt}$$

$$v = \omega L I_m \left[-\sin\left(\omega t + \theta_i\right)\right]$$

$$v = \omega L I_m \sin\left(-\omega t - \theta_i\right)$$

$$v = \omega L I_m \cos\left(\omega t + \left[\theta_i + 90^\circ\right]\right)$$

$$\bar{V} = \omega L I_{RMS} \angle \theta_i + 90^\circ$$

$$\bar{S}_L = \omega L I_{RMS}^2 \angle 90^\circ$$

$$\text{PF} = 0 \text{ lag}$$

$$i = C\frac{dv}{dt}$$

$$i = \omega C V_m \left[-\sin\left(\omega t + \theta_v\right)\right]$$

$$i = \omega C V_m \sin\left(-\omega t - \theta_v\right)$$

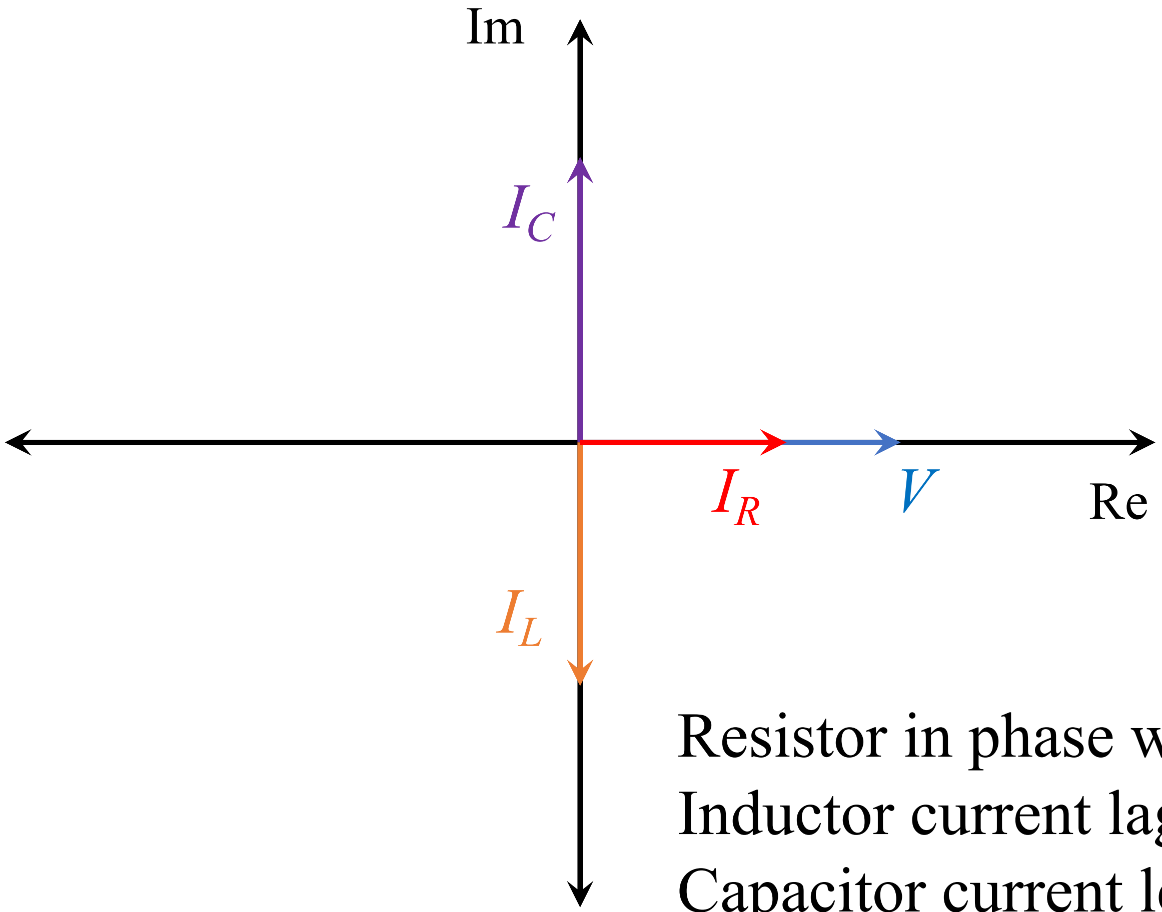
$$i = \omega C V_m \cos\left(\omega t + \left[\theta_v + 90^\circ\right]\right)$$

$$\bar{I} = \omega C V_{RMS} \angle \theta_v + 90^\circ$$

$$\bar{S}_C = \omega C V_{RMS}^2 \angle -90^\circ$$

$$\text{PF} = 0 \text{ lead}$$

Plotting in Complex Plane



Resistor in phase with voltage
Inductor current lags voltage by 90°
Capacitor current leads voltage by 90°